Empirical Project

Replication of *Returns to Scale in Electricity Supply*

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1 Introduction

In 1963, Mac Nerlove set out to determine the returns to scale in the the electricity industry. As Hayashi points out, this paper illustrates the need for econometric analysis of data as opposed to just taking the data at face value. For example, the data may give an impression of increasing returns to scale when plotted. Using econometrics can help us to get a better grasp of the relationship between variables. In this paper, I will use the same data set used by Nerlove to attempt to recreate his results.

1.1 Data

The data set comes from Hayashi's website, and is identical to the one Nerlove used in his analysis. The data are a 1955 cross-section of 145 firm in 44 states. The data set contains the total cost incurred by the firm, the output (in kilowatt hours), and the price of three factor inputs: labor, fuels, and capital. I then transformed the data by taking the natural log of each obnersrvation. This step will allow the use of a linear model. Nerlove explains that prices for labor and fuel were readily available, but finding data for capital prices proved to be more challenging. Nerlove needed to estimate the price of capital, and did this by "taking interest and depreciation charges on the firm's entire production plant and multiplying by the ratio of the value of steam plant to total plant as carried on the firm's books" (Nerlove, p. 190). He explains that this isn't a perfect way to estimating the price of capital, but it does give a reasonable estimation. Further explanation of the data can be found in Appendix B of Nerlove's article.

Hayashi argues that the data meet the requirements for the OLS assumptions to hold (Hayashi, p. 64). We can fairly easily assume that the explanatory variables for one firm are unrelated to the error term for another firm. In other words, $\mathbf{x}_i$ is uncorrelated with ϵ_j when $i \neq j$. Because we are assuming that the electrical industry is a perfectly competitive one, each firm takes the price as given by the market. That includes the price that the firm is selling electricity for (i.e. they don't act like a monopoly), but also the price that the firm pays for its factor inputs (i.e. they don't act like a monopsony). An individual firm can't dictate the price it pays for, say, coal because the coal suppliers can sell elsewhere for a higher price. If this is true, then we don't need to worry about any correlation between factor prices and the error term for any particular firm. Hayashi then makes the argument that a firm's output is also uncorrelated with the firm's error term (Hayashi, p. 64). If it is true that a firm takes demand as given and meets the demand, then output and the error term are uncorrelated.

2 Model

Nerlove starts out with a simple Cobb-Dougls model, transforming it with a natural log to get a liner model (using Hayashi's notation):

$$\log(TC_i) = \beta_1 + \beta_2 \log(Q_i) + \beta_3 \log(p_{i1}) + \beta_4 \log(p_{i2}) + \beta_5 \log(p_{i3}) + \epsilon_1 \tag{1}$$

where TC_i is firm i's total cost, Q_i is firm i's output, and p_{i1}, p_{i2}, and p_{i3} are firm i's price of labor, capital, and fuels, respectively. The problem with this model is that it doesn't restrict the coefficients of the prices to sum to one. If this condition doesn't hold, we violate the assumptions of the cost function. To remedy this, Nerlove subtracts the log of one of the factor prices – he picks the price of fuel, but argues that it doesn't make any difference either economically or statistically which price is chosen (Nerlove p. 175) – off of each side of the equation,s creating the following model:

$$\log\left(\frac{TC_i}{p_{i3}}\right) = \beta_1 + \beta_2 \log(Q_i) + \beta_3 \log\left(\frac{p_{i1}}{p_{i3}}\right) + \beta_4 \log\left(\frac{p_{i2}}{p_{i3}}\right) + \epsilon_i \tag{2}$$

This means that estimated coefficients for the prices of fuel and labor are given by the OLS estimation. The coefficient for the price of capital can be calculated through our restriction that $\beta_3 + \beta_4 + \beta_5 = 1$. Since we know β_3 and β_4, we can easily find β_5.

3 Results

First I run the unrestricted OLS model from equation (1). My results are summarized in the following table:

Variable	Coefficient	Standard Error
(Constant)	-3.526^*	1.774
Output	0.72^*	0.017
p_{i1}	0.436	0.291
p_{i2}	-0.219	0.229
p_{i3}	0.426^*	0.1

$R^2 = 0.925 \qquad SSR = 21.55$

*=significant at the 5% level

My results match the results found in Nerlove's paper. Nerlove argues that the reason the coefficient for the price of capital is wrong is that there are problems with the measurement of the data. As was discussed above, the measurement for capital isn't ideal, and that may very well cause the estimation of the β_4 to be wrong. To find the degree of returns to scale from the OLS estimations, we divide one by the estimate of β_2. That gives us: $r = \frac{1}{\beta_2} = \frac{1}{0.72} = 1.4$

Next, I ran the restricted model from equation (2). My results once again mirrored Nerlove's:

Variable	Coefficient	Standard Error
(Constant)	-4.69^*	0.88
Output	0.72^*	0.017
p_{i1}/p_{i3}	0.593^*	0.204
p_{i2}/p_{i3}	-0.007	0.19

$R^2 = 0.931 \qquad SSR = 21.64$

*=significant at the 5% level

Once again, the measurement problems surrounding the price of capital are influencing the results. It would be very hard to argue that, in either regression, the price of capital has no statistically significant impact on costs.

We can use both the restricted and unrestricted OLS results to test whether or not our homogeneity restriction holds. That is, $H_0 : \beta_3 + \beta_4 + \beta_5 = 1$. The F-Stat formula gives us:

$$F = \frac{(SSR_r - SSR_u)/q}{SSR_u/(n-K)} = \frac{(21.64 - 21.55)/1}{21.55/(145-5)} = 0.57 \tag{3}$$

The numerator degrees of freedom is one and the denominator degrees of freedom is 140. With an F-Stat of 0.57 and a critical value of 3.9, we can comfortably fail to reject H_0. This means that homogeneity holds.

We can also use our results from the restricted regression to test whether or not there is constant returns to scale in the industry. Because $\beta_2 = \frac{1}{r}$, there are constant returns to scale if and only if $\beta_2 = 1$, which implies that $r = 1$. Using a simple t-test, we get:

$$t = \frac{0.72 - 1}{0.017} = -16 \tag{4}$$

A t-ratio of -16 is clearly going to be in the rejection region. Thus, we reject the null and conclude that there is not constant returns to scale in the electricity industry.

Finally, we can determine what happens to a firm's return to scale as its size grows. Following Nerlove, I split the data up by firm size into five equal parts (each with 29 firms). Running the restricted OLS regression for each of the five sub-sets gives us five β_2 numbers. Remember from above, we get the returns to scale by dividing 1 by the coefficient. The following table summarizes the information:

Regression	Coefficient	Returns to Scale
1	0.4	2.5
2	0.65	1.53
3	0.93	1.07
4	0.91	1.1
5	1.04	0.96

This means that the returns to scale decreases as the size of the firm increases. Nerlove also found this when he ran these test in his analysis. Basically, it means that larger firms have a harder time increasing output when increasing input. All but the largest of the firms are found to have increasing returns to scale. It's not terribly surprising that returns to scale drop off as size increases. A larger firm is going to have a harder time of increasing output than a small firm. These results should also be reliable because they don't rely on the price of capital, which was our most problematic variable.

4 Conclusion

I learned a lot while doing this paper. I learned about OLS regression techniques like using the ratios of variables to create a restricted regression. I also learned how it is possible to test for thing like returns to scale using OLS. I found it very interesting how we were able to use an F-test to conclude that our coefficients do indeed sum to one (meeting one of our assumptions), and a t-test to conclude that we do not have constant returns to scale.

I also learned about using econometric tools to analyze an industry. Even just replicating someone else's paper allowed me to get a better picture of what it takes to get a grasp of a particular market. I hope that I can use the econometric tools I've learned in this class to help me when I am working at a firm, and doing an analysis like this one is a good practice.

5 Bibliography

Hayashi, Fumio. Econometrics. New York: Princeton UP, 2001.

Nerlove, Marc. "Returns to Scale in Electricity Supply." Ed. C. Christ. Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld. Stanford UP, 1963.